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**MOBILITY POWER FLOW ANALYSIS
OF COUPLED PLATE STRUCTURE
SUBJECTED TO MECHANICAL
AND ACOUSTIC EXCITATION**

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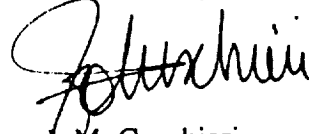
**Final Report
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FOREWORD

This report describes the work performed between September 1988 to January 1991 under Research Grant Numbers NAG-1-685, entitled, "Use of Energy Accountancy and Power Flow Techniques for Aircraft Noise Transmission" and Grant Number NAG-1-1077, entitled "Mobility Power Flow Analysis of Flexible Plate Structure Enclosing an Acoustic Cavity". Other progress reports have been submitted under these research grants which have already been published as NASA Contract reports. During the phase of this work, a Ph.D. Thesis was completed with the title, "Vibrational Power Flow in Thick Connected Plates", the Abstract of which will be published in the Journal of the Acoustical Society of America in the Technical Notes and Research Briefs section. The work in this thesis was also partially funded by an ONR Fellowship which paid for the stipend of the graduate student. Two Master's thesis have also been completed during this period. This report deals with a generalization of the mechanical loading problem and on the use of the mobility power flow approach to deal with the fluid structure interaction problem. A case study of the L-shaped plate will be considered. The final part of this report deals with the coupling both acoustic and structural of multiple coupled plates.

The author would like to acknowledge the graduate students who assisted in the work on this project, the Department of Ocean Engineering and most importantly the financial support by the Structural Acoustics Branch of the NASA Langley Research Center.

Submitted by



L. M. Cuschieri

Principal Investigator

ABSTRACT

This report generalizes the mobility power flow approach, previously applied in the derivation of expressions for the vibrational power flow between coupled plate substructures forming an L configuration and subjected to mechanical loading. Using the generalized expressions, both point and distributed mechanical loads on one or both of the plates can be considered. The generalized approach is extended to deal with acoustic excitation of one of the plate substructures. In this case the forces (acoustic pressures) acting on the structure are dependent on the response of the structure because of the scattered pressure component. The interaction between the plate structure and the acoustic fluid leads to the derivation of a corrected mode shape for the plates' normal surface velocity and also for the structure mobility functions. The determination of the scattered pressure components in the expressions for the power flow represents an additional component in the power flow balance for the source plate and the receiver plate. This component represents the radiated acoustical power from the plate structure. For a number of coupled plate substructures, the acoustic pressure generated by one substructure will interact with the motion of another substructure. That is, in the case of the L-shaped plate, acoustic interaction exists between the two plates substructures due to the generation of the acoustic waves by each of the substructures. An approach to deal with this phenomena is described in this report.

1. INTRODUCTION

Mobility Power Flow (MPF) methods [1-5] have previously been shown to provide an effective structure analysis tool to deal with the response of one and two dimensional structures with direct mechanical excitation. Comparison of the MPF results with experimental results on a one dimensional structure [3], and with numerical (Finite Element Analysis (FEA)) [2] and statistical (Statistical Energy Analysis (SEA)) [4] results, showed good agreement. Furthermore, using the MPF method, the exchange of vibrational power between the substructures can be obtained for different structural wave components [6].

One limitation of past work [1-6] is that, only excitation by direct point forces has been considered. The interaction with the surrounding acoustic medium has been neglected. In the case of excitation of an aircraft fuselage, distributed loading on the surface of a panel from acoustic loading can be as important as the excitation from directly applied forces at defined locations on the structure. The contribution to the overall behavior of the structure from acoustic distributed loading would be significant when the influence of the surrounding acoustic medium is not negligible.

The main objective of this report is to extend the MPF method to distributed excitation. Two types of loading conditions are considered, a distributed mechanical force load of arbitrary shape and a distributed load created by an obliquely incidence acoustic waves on the surface of one of the plate substructures. The difference between these two types of distributed loads is that in the mechanical loading case the applied force is independent of the response of the structure, while in the case of the excitation from an incident acoustic wave, the loading is dependent on the response of the structure due to the presence of the scattered acoustic waves. In this latter type of loading coupled expressions must be developed for the excitation and response. To be consistent with previous work the structure that will be considered in the analysis is an L-shaped plate. In the case of acoustics loading, an acoustic wave is assumed incident on one side of one of the flat plate substructures.

In using the MPF method [1], the structure is modeled by a series of coupled substructures with each substructure analyzed independently. The coupling between the substructures is defined through the boundary conditions at the junctions, taking into account the forces and moments that the substructures exert on each other. Expressions for the input power to a substructure and for the transferred power between the substructures, are obtained in terms of the input and junction velocity contributions and the forces and moments that are applied on the substructures. The junction forces and velocities are described in terms of the substructures' mobility functions.

With direct force excitation of the structure, the input and transferred power can be written in terms of the input and transfer structural mobility functions of the substructures [2]. These only depend on the geometry of the specific substructure and on the frequency of excitation. The solution for the response of the global structure can be presented in a matrix form, which allows the method to deal with a large number of connected substructures. One advantage of dividing the global structure into subsystems is that, if the dependence of the response of the global structure on one of the subsystems is required, it is not necessary to repeat the whole of the

analysis for the global structure, but only for that part that deals with the modified subsystem. Furthermore, the analysis is efficient to implement if the subsystem elements are identical, in which case the independent response of only a typical subsystem is required.

The first section of this report deals with distributed mechanical excitation. In the following section, the analysis is extended to deal with acoustic excitation. A harmonic plane acoustic wave is assumed to be incident on one side of one of the flat plate substructures. The excitation will be considered to be on the outside surface, and the plate being excited is located in an infinite baffle. This is to avoid the edge effects. With acoustic excitation, the effects of fluid loading are included in the analysis. The scattered pressure will interfere with the incident pressure field and the mode shape of the structure will be influenced by the presence of the fluid. The fluid that will be considered here is air and therefore approximations applicable for light fluid loading can be applied.

The analysis for acoustic excitation is then extended in the following section to consider the interaction of the scattered acoustic waves from different sections of the L-shaped plate structure. For coupled structural surfaces surrounded by an acoustic medium, the acoustic waves generated by the motion of one of the substructure surfaces will interact with the motion of other substructures. That is, in this case, coupling between the substructural elements of the structure is both structural and acoustical.

To investigate the problem of the acoustic interaction between coupled structures, the acoustic field on the inside surface between the two coupled plates joined in the L-shaped configuration is considered. The reason for selecting the inside surface and not the outside is to avoid the sharp edge at the boundary between the two plates. The influence of this sharp edge on the acoustic field, especially its diffraction characteristics, can be very complex to model. The influence of the second structural component, is included by replacing with a rigid baffle. Due to the presence of the second plate, and its influence on the acoustic field, the effective surface area of the first component is twice the actual surface area, and the motion of the surface of this component is symmetrical about the edge bounded by the rigid baffle which is in a plane perpendicular to this surface. The baffle acts as a reflecting, zero velocity, surface. In this case a complete solution is sought for both plate substructures from consideration of compatibility of motion and forces at the junction, including the modification of the acoustic field.

2. DISTRIBUTED MECHANICAL LOADING

The development of power flow and mobility expressions for the distributed force loading, with arbitrary dependence on the x and y coordinates, is very similar to the work reported previously for point loading [2,4]. The differences are that the presentation of the mobility functions can be generalized to apply for any loading condition.

Using the configuration and axes definition as shown in figure (1), the modal input mobility function for the junction, and the transfer mobility function between the junction and any point on the plate surface, both functions being defined as the response per unit applied moment for every mode m along the junction, are given by [2,4].

$$M_{2m} = M_{3m} = \frac{\theta_m}{T_m} = \frac{j}{2\sqrt{(\rho h D^*)}} \left[\frac{k_2}{\tan(k_2 b)} - \frac{k_1}{\tanh(k_1 b)} \right] \quad (1)$$

$$M_{12m}(y) = \frac{\dot{u}_m(y)}{T_m} = \frac{j}{2\sqrt{(\rho h D^*)}} \left[\frac{\sin(k_2 y)}{\sin(k_2 b)} - \frac{\sinh(k_1 y)}{\sinh(k_1 b)} \right] \quad (2)$$

where T_m is the mode m component of the edge moment, k_1 and k_2 are defined by:

$$k_1^2 = 2k_x^2 + k_y^2 ; \quad k_2 = k_y$$

$$k_x = m\pi/a ; \quad k_y^2 = \omega \sqrt{\frac{\rho h}{D^*}} - k_x^2 \quad (3)$$

u_m is the mode m component of the plate surface displacement, θ_m is the mode m component of the angular displacement at the edge of the plate and D^* and ρh are the plate flexural rigidity and surface density respectively. In the above equations it is assumed that the two plate substructures are identical. m and n are respectively the mode numbers for the x and y directions, that is m is along the junction and n perpendicular to the junction, and a and b are the dimensions of the plates (figure 1). Note that in equation (2) the transfer mobility is a function of coordinate y . This is necessary since the load is distributed and defined everywhere along the y direction on the plate surface.

The above modal mobility expressions are for a plate subsystem with an applied edge moment. This is one of the configuration subsystems for which mobility expressions are necessary [2]. The second configuration considered is the one which represents an external distributed load. The distributed load can be decomposed into its modal components, F_m and F_n as follows:

$$F(x, y) = F_0 f(x) f(y) \quad (4)$$

$$F_m = \frac{2}{a} \int_0^a F_0 f(x) \sin\left(\frac{m\pi x}{a}\right) dx \quad (5)$$

$$F_n = \frac{2}{b} \int_0^b f(y) \sin \left[\frac{m\pi y}{b} \right] dy \quad (6)$$

$$F_{mn} = F_m F_n \quad (7)$$

Using this load decomposition, mobility functions for the second plate configuration can be derived in terms of modal mobilities. The modal mobility for the input distributed load, M_{1mn} defined as the ratio of the mode (m,n) component of the transverse velocity of the surface of the plate, to the (m,n) component of the applied distributed surface load, is given by:

$$M_{1mn} = \frac{\dot{u}_{mn}}{F_{mn}} = \frac{jf}{2\pi\rho h} \frac{1}{\left[f_{mn}^* \right]^2 - f^2} \quad (8)$$

where

$$f_{mn}^* = \frac{1}{2\pi} \sqrt{\frac{D}{\rho h} \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right]} \quad (9)$$

and u_{mn} is the mode m,n component of the plate surface displacement. The expression for the transfer mobility, M_{21m} defined as the ratio of the mode m component of the plate edge angular velocity, to the mode m component of an applied distributed load on the surface of the plate is given by;

$$M_{21m} = \frac{\dot{\theta}_m}{F_m} = \frac{jf}{2\rho hb} \sum_n \frac{(-1)^n n F_n}{\left[f_{mn}^* \right]^2 - f^2} \quad (10)$$

The evaluation of the above mobility functions is obtained from consideration of the equations of motion for a plate structure.

The expression for the input and transmitted power are the same as in the case of point loading [2] except that all the mobility functions are changed to modal expressions:

$$\text{Power}_{\text{input}} = \frac{1}{2} \text{Real} \left\{ \sum_{m,n} \frac{ab}{4} |F_{mn}|^2 M_{1mn} - \frac{a}{2} \sum_m \frac{M_{21m}}{M_{2m} + M_{3m}} |F_m|^2 \int_0^b f(y)^* M_{12m}(y) dy \right\} \quad (11)$$

$$\text{Power}_{\text{trans}} = \frac{1}{2} \frac{a}{2} \sum_m \left| \frac{M_{21m} F_m}{M_{2m} + M_{3m}} \right|^2 \text{Real} [M_{3m}] \quad (12)$$

For a point load, the force in this case can be described mathematically by:

$$F(x,y) = F_0 \delta(x-x_0) \delta(y-y_0) \quad (13)$$

and

$$F_{mn} = \frac{4}{ab} F_0 \sin \left[\frac{m\pi x_0}{a} \right] \sin \left[\frac{n\pi y_0}{b} \right] \quad (14)$$

where x_0 and y_0 are the x and y coordinates of the point of application of the load. If the load is applied at the center of the plate, ($x_0 = a/2$ and $y_0 = b/2$), then only odd values for m and n are allowed. Substituting equation (14) into equations (11) and (12), the results obtained for the input and transferred power, when the load is in the center of the source plate, are shown in figure (2). Comparing these results to those obtained in reference [3], the two sets of results are identical. This verifies the formulation of the power flow expressions in the form shown in equations (11) and (12).

For a uniform distributed load described by:

$$F(x,y) = F_0 \quad (15)$$

$$f(y) = 1.0 \quad (16)$$

and

$$F_{mn} = \frac{16 F_0}{\pi^2 (2m + 1) (2n + 1)} \quad (17)$$

the solution for the input and transferred power is obtained using the same equations (11) and (12). The results for the power flow are shown in figure (3).

Comparing the results for the power flow obtained for the point load at the center of the plate with the results obtained for the uniformly distributed load, the following similarities and differences can be observed. First, the modes excited by the two types of loading are identical. This is expected since both loading conditions are symmetrical about the center of the source plate. Second, the general level of the power flow for the distributed load case decreases with frequency, while that for the point load does not decrease. The reason for this can be mathematically described by the inverse dependency of the modal components of the distributed applied load on the mode number (equation (17)). Physically this implies that the high frequency modal components of the load are suppressed. The results for the distributed load would be similar to those for excitation by normal incidence acoustic plane waves, if the scattered pressure component is neglected.

3. EXCITATION BY INCIDENT ACOUSTIC WAVES

3.1. GENERAL APPROACH

In dealing with distributed excitation from incident acoustic waves, the scattering of the incident acoustic wave due to the response of the receiving structure must be included in the analysis [7]. The scattered acoustic field is a function of the response of the structure, and therefore, the loading on the structure consists of two components; one component is the incident pressure being blocked by the surface and the second component is the pressure from the scattered acoustic waves.

The component associated with the blocked incident pressure is independent of the motion of the structure except for the phase differences between structure response and surface motion. The component associated with the scattered pressure is a direct function of the motion of the structure and therefore the loading becomes structure response dependent. In dealing with this type of distributed loading, some modifications to the basic formulation of the expressions for the vibrational power input and power flow are required. To demonstrate the differences in the formulation, the case of a point load, which is not independent of the response velocity, applied at a point on an arbitrary structure is discussed.

3.2. RESPONSE DEPENDENT POINT LOADING

Consider the configuration shown in figure (4), where the input load F is a function of the response velocity. The governing equations of motion for this configuration are [1];

$$V = F M_1 + F_A M_{12} \quad (18)$$

$$V_A = -F_A M_3 \quad (19)$$

$$= F_A M_2 + F M_{21} \quad (20)$$

but

$$F = A + BV \quad (21)$$

where A and B are two constants. All the other terms in the above expressions have the same definition as in previous work [1]. Equations (19) and (20) can be used to solve for the junction force F_A in terms of the input load F, and then substitute into equation (18) to solve for the velocity V. In the case where F is a function of V, the result for V must be substituted back into the expression for F and then used to solve for F_A and V_A . Alternatively V and F_A are expressed in terms of the constants A and B and the mobility functions. That is,

$$\begin{bmatrix} (1 - BM_1) & -M_{12} \\ BM_{21} & M_2 + M_3 \end{bmatrix} \begin{bmatrix} V \\ F_A \end{bmatrix} = \begin{bmatrix} A M_1 \\ -A M_{21} \end{bmatrix} \quad (22)$$

and

$$\begin{bmatrix} V_A \end{bmatrix} = \begin{bmatrix} -M_3 \end{bmatrix} \begin{bmatrix} F_A \end{bmatrix} \quad (23)$$

The input and transmitted power components, P_i and P_t respectively, are given by:

$$\begin{bmatrix} P_i \\ P_t \end{bmatrix} = \begin{bmatrix} (A + BV) & F_A \end{bmatrix} \begin{bmatrix} V^* \\ V_A^* \end{bmatrix} \quad (24)$$

In this case a slightly different set of matrix equations are obtained, which can be solved for the evaluation of the power input and power flow between the subsystems. This result shows that the same general approach can be used when the external loading on a structural component is influenced by the response of the structure. The use of this result in the case of acoustic excitation will be described in the following subsection.

3.3. ACOUSTIC EXCITATION

Consider a structure in the plane (x,y,0), with an acoustic fluid occupying the half-space $z > 0$ (figure 5). A plane wave incident on the surface of this structure can be described by:

$$p_i(x,y,z) = P_0 e^{-jk_x \cos \theta \sin \phi} e^{-jk_y \sin \theta \sin \phi} e^{kz \cos \phi} \quad (25)$$

The direction of the plane wave incident on the surface of the structure subtends an angle ϕ to the normal of the plate and an angle θ to the x-axis direction in the plane of the structure

surface (figure 5). The response of the structure to the incident acoustic wave creates an acoustic field which will interact with the incident field. The total pressure acting on the structure surface is thus modified due to the presence of the scattered acoustic pressure component. The total pressure acting on the surface of the structure can be represented by [7]:

$$p(x, y, z=0) = 2p_i(x, y, z=0) + p_s(x, y, z=0) \quad (25)$$

where $p_i(x, y, z=0)$ represents the incident pressure component and $p_s(x, y, z=0)$ represents the scattered pressure component. The factor of 2 in front of the incident pressure component is introduced to take into account the reflected pressure component which is equal and opposite in the z direction to the incident pressure component (twice $p_i(x, y, z=0)$ represents the blocked pressure component).

From equation (26) it can be observed that the scattered pressure component can be dealt with separate from the incident pressure. If the structure has responses which are associated with different forces and moments acting on the structure, each of these responses creates a scattered pressure component. The total scattered pressure would have contributions from these different components, which can be dealt with separately.

In the case of the L-shaped plate, the scattered pressure from the source plate, the plate receiving the acoustic wave, has two components, one which can be associated with the acoustic excitation and another component which can be associated with the edge moment representing the influence of the attached receiver plate. Each of these components is dealt with separately, provided the changes in the response of the structure due to the fluid loading are taken into account. This approach fits directly within the MPF method, since with the MPF the global structure response is analyzed in terms of the separate responses of the subsystems representing the global structure. The scattered pressure components of each subsystem is thus separately determined and then summed to compute the power input to the global structure.

If the structure response mode shape is apriori known, the influence of the fluid loading on the structure can be directly determined from a solution of the coupled equations of motion [7]. However, this approach is not applicable when the mode function is unknown. An alternative approach has been suggested by Leppington [8], where solutions for the structural response and scattered pressure are obtained based on an approximate solution for the case where the scattered pressure is neglected. The complete solution is given in terms of a correction factor, introduced to take into account the scattered pressure component and still satisfy the boundary conditions for the structure and the acoustic medium. The correction factor is independent of the magnitude of the applied loading and is mainly controlled by the boundary conditions for the structure and the type of loading. Once the structural response is known, the scattered pressure is computed, and used to determine the pressure loading on the surface of the structure and the vibrational power input to the structure.

For the L-shaped plate, the response, including the correction factor, for each of the two plate subsystems, one representing a simply supported plate with an incident acoustic wave and the other representing a simply supported plate with an edge moment will be separately

determined from which mobility functions can be obtained. The total response of the structure and the scattered pressure components are computed by combining these mobility functions which already include the influence of the fluid loading (scattered pressure) for the type of loading considered in the determination of the mobility terms. Similarly for the power flow components.

3.4. SIMPLY SUPPORTED PLATE

The equation of motion for the transverse displacement of a plate is given by [9]:

$$D^* \nabla^4 u(x, y) - \rho \omega u(x, y) = p(x, y, z=0) \quad (27)$$

where $p(x, y, z=0)$ represents the pressure loading on the plate. This term consists of the incident pressure component plus the scattered pressure component (equation 26).

Assuming a sinusoidal mode shape in the x direction [2], the direction parallel to the junction, the displacement $u(x, y)$ can be written in the form,

$$u(x, y) = \sum_{m=1}^{\infty} u_m(y) \sin\left[\frac{m\pi x}{a}\right] \quad (28)$$

Substituting this equation into equation (27),

$$\left[\frac{m\pi}{a}\right]^4 u(y) - 2\left[\frac{m\pi}{a}\right]^2 \frac{\partial^2 u(y)}{\partial y^2} + \frac{\partial^4 u(y)}{\partial y^4} - k_p^4 u(y) = \frac{p(y)}{D_p} \quad (29)$$

where the subscript m is dropped from $u_m(y)$ for simplicity, k_p is the plate wavenumber and $p(y)$ is defined by

$$p(y) = \frac{2}{a} \int_0^a p(x, y) \sin\left[\frac{m\pi x}{a}\right] dx \quad (30)$$

Substituting for $p(x, y)$,

$$p_i(y) = \frac{2P_o}{a} \Gamma_1 e^{(-jk_y \sin\theta \sin\phi)} \quad (31)$$

where

$$\Gamma_1 = \frac{m\pi}{a} \frac{(-1)^m e^{-jk a \cos \theta \sin \phi}}{(k \cos \theta \sin \phi)^2 - (m\pi/a)^2} \quad (32)$$

The scattered pressure component for mode m is a function of the normal surface velocity and is therefore dependent on the solution for the response of the plate. A coupled solution for the plate and the scattered pressure is required.

If the scattered pressure component is neglected, the right hand-side of equation (27) has only one component, given by equation (31) multiplied by a factor of 2. In this case the solution for the displacement of the plate can be written in the form:

$$u(y) = u_p(y) + u_h(y) \quad (33)$$

where $u_p(y)$ is the particular solution of equation (27), given by:

$$u_p(y) = \frac{4P_o}{a} \frac{\Gamma_1}{D^*} \frac{e^{-jk y \sin \theta \sin \phi}}{[(m\pi/a)^2 + (k \sin \theta \sin \phi)^2]^2 - k_p^4} \quad (34)$$

and $u_h(y)$ is the solution of the homogeneous equation of motion, given by

$$u_h(y) = A \cosh(k_1 y) + B \sinh(k_1 y) + C \cos(k_2 y) + D \sin(k_2 y) \quad (35)$$

where k_1 and k_2 are defined by equation (3).

The coefficients A, B, C and D in equation (35) are selected to satisfy the boundary conditions along the y direction. In the case of the simply supported plate these boundary conditions are:

$$\begin{aligned} u(y=0) &= 0 ; \quad \frac{\partial^2 u(y=0)}{\partial y^2} = 0 \\ u(y=b) &= 0 ; \quad \frac{\partial^2 u(y=b)}{\partial y^2} = 0 \end{aligned} \quad (36)$$

Solving for A,B,C and D using these boundary conditions yield

$$\begin{aligned}
 A &= - \frac{u_p''(0) + u_p(0)k_2^2}{2k_p^2} \\
 B &= \frac{-k_2^2 u_p(b) - u_p''(b) + [u_p''(0) - u_p(0)k_2^2] \cosh(k_1 b)}{2k_p^2 \sinh(k_1 b)} \\
 C &= \frac{u_p''(0) - u_p(0)k_1^2}{2k_p^2} \\
 D &= \frac{-k_1^2 u_p(b) + u_p''(b) - [u_p''(0) - u_p(0)k_1^2] \cos(k_2 b)}{2k_p^2 \sin(k_2 b)}
 \end{aligned} \tag{37}$$

where the double prime indicates a second derivative with respect to y .

To compensate for neglecting the scattered pressure component, a corrective complex coefficient K_I is introduced [8],

$$u_1(y) = u_p(y) + u_{h1}(y) + K_I u_{h2}(y) \tag{38}$$

where

$$u_{h2} = \sin(k_2 y) \tag{39}$$

The main influence of the scattered pressure component will be on the contribution from the solution to the homogenous equation of motion [8], especially near resonance when $k_2 = (n\pi)/b$. This is the reason why the correction factor K_I is only applied to the term $\sin(k_2 y)$. The complex correction coefficient K_I will be close to the value of the coefficient D away from the resonant frequencies and will deviate from this value at the resonant frequencies where the influence of the fluid on the velocity mode shape is most significant [8]. If $u_1(y)$ represents the exact solution for the response of the simply supported plate to the acoustic excitation, then the approximate result obtained when neglecting the scattered pressure component introduces an error $u_e(y)$ given by

$$u_e(y) = u(y) - u_1(y) \tag{40}$$

By substituting the above equation into the equation of motion (equation 27), the error term $u_e(y)$ must satisfy the equation,

$$\left[\frac{m\pi}{a} \right]^4 u_e(y) - 2 \left[\frac{m\pi}{a} \right]^2 \frac{\partial^2 u_e(y)}{\partial y^2} + \frac{\partial^4 u_e(y)}{\partial y^4} - k_p^4 u_e(y) = \frac{p_s(y)}{D^*} \quad (41)$$

In solving equation (41), a set of boundary conditions that apply to $u_e(y)$ can be derived from the boundary conditions for $u(y)$. These are given by,

$$u_e(y=0 \text{ or } b) = -u_l(y=0 \text{ or } b) = -(u_g(0 \text{ or } b) + K_I u_h(0 \text{ or } b)) = 0 \quad (42)$$

and

$$\begin{aligned} \frac{\partial u_e^2(y=0 \text{ or } b)}{\partial y^2} &= - \frac{\partial u_l^2(y=0 \text{ or } b)}{\partial y^2} \\ &= - \left[\frac{\partial u_g^2(y=0 \text{ or } b)}{\partial y^2} + K_I \frac{\partial u_h^2(y=0 \text{ or } b)}{\partial y^2} \right] = 0 \end{aligned} \quad (43)$$

A solution for $u_e(y)$ is derived in terms of the Green function $G(\xi, y)$ for the plate structure [8]

$$\begin{aligned} u_e(y) = \frac{1}{D^*} \int_0^b & G(\xi, y) p_s(\xi) d\xi - G'(b, y) u_e''(b) + G'(0, y) u_e''(0) \\ & - G'''(b, y) u_e(b) - G'''(0, y) u_e(0) \\ & + 2(m\pi/a)^2 [G'(b, y) u_e(b) - G'(0, y) u_e(0)] \end{aligned} \quad (44)$$

where the primes for the function G represent derivatives with respect to ξ . The function $G(\xi, y)$ is defined by:

$$\begin{aligned}
G(y, \xi) = & - \frac{\sinh[k_1(\xi-b)] \sinh[k_1 b]}{2k_p^2 k_1 \sinh[k_1 b]} + \frac{\sin[k_2(\xi-b)] \sin[k_2 b]}{2k_p^2 k_2 \sin[k_2 b]} \quad y \leq \xi \\
& - \frac{\sinh[k_1 b] \sinh[k_1(\xi-b)]}{2k_p^2 k_1 \sinh[k_1 b]} + \frac{\sin[k_2 b] \sin[k_2(\xi-b)]}{2k_p^2 k_2 \sin[k_2 b]} \quad y > \xi
\end{aligned} \tag{45}$$

which is obtained from the solution to the homogeneous equation of motion.

From the expression for $u_e(y)$, the coefficient K_I is selected such that $u_e(y) = 0$ for all values of y . Substituting equation (45) into equation (44), and equating $u_e(y)$ to zero, an expression is obtained that contains $p_s(\xi)$, which is still an unknown. However, $p_s(\xi)$ can be expressed as a function of the normal displacement of the plate, by using the momentum equation. Introducing this substitution and after some manipulations which are presented in the appendix, the following result is obtained for $p_s(\xi)$

$$\frac{p_{s_m}}{D^*} = \frac{j}{2\pi^2 a} \left[\frac{m\pi}{a} \right]^2 \alpha_1 \int_{-\infty}^{\infty} u_{1_m}(\beta) I_m(\beta) e^{-j\beta\xi} d\beta \tag{46}$$

where α_1 is the fluid loading coefficient, $\alpha_1 = (\omega^2 \rho_0)/D_p$, ρ_0 is the fluid density, and β is the spatial Fourier transform variable in the y direction. $I_m(\beta)$ is defined in the appendix. From equations (38), (39), (42), (43), (44) (45) and (46) an expression for the correcting coefficient K_I is derived,

$$K_I = \frac{\frac{-j}{2\pi^2 a} \left[\frac{m\pi}{a} \right]^2 \alpha_1 \int_{-\infty}^{\infty} u_p(\beta) + u_{h_1}(\beta) I(\beta) S(\beta) d\beta + \text{RES}_1}{\frac{-j}{2\pi^2 a} \left[\frac{m\pi}{a} \right]^2 \alpha_1 \int_{-\infty}^{\infty} u_{h_2}(\beta) I(\beta) S(\beta) d\beta + \text{RES}_2} \tag{47}$$

where

$$S(\beta) = \frac{e^{-j\beta b} - \cos(k_2 b)}{\beta^2 - k_2^2} \tag{48}$$

$$RES_1 = \begin{bmatrix} k_1^2 + k_2^2 \end{bmatrix} \begin{bmatrix} \cos(k_2 b) u_p(b) - u_p(0) \end{bmatrix} \quad (49)$$

and

$$RES_2 = (k_1^2 + k_2^2) \sin(k_2 b) \quad (50)$$

Having solved for the response of the simple supported plate a modal mobility function can be defined by the ratio of the modal velocity response to the input incident pressure. That is, using the same notation as in previous reports, with the subscript 1 to represent the input location, the input modal mobility for mode m is given by,

$$M_{1m}(y) = \frac{j\omega u_1(y)}{P_0} = \frac{j\omega 4\Gamma_1}{aD^*} \frac{e^{(-jk_a y)}}{\Xi} + j\omega [A' \cosh(k_1 y) + B' \sinh(k_1 y) + C' \cos(k_2 y) + D' \sin(k_2 y)] + j\omega K_I \sin(k_2 y) \quad (51)$$

where

$$k_a = k \sin \theta \sin \phi \quad (52)$$

$$A' = \frac{2\Gamma_1}{aD^* k_p^2} \frac{[k_2^2 - k_a^2]}{\Xi} \quad (53)$$

$$B' = \frac{2\Gamma_1 (k_2^2 + k_a^2)}{aD^* k_p^2} \left[\frac{e^{-jk_a b} + \cosh(k_1 b)}{\Xi \sinh(k_1 b)} \right] \quad (54)$$

$$C' = \frac{-\Gamma_1}{aD^* k_p^2} \frac{[k_1^2 + k_a^2]}{\Xi} \quad (55)$$

$$D' = \frac{\Gamma_1(k_1^2 + k_a^2)}{aD^*k_p^2} \left[\frac{-e^{-jk_a b} + \cos(k_2 b)}{\Xi \sin(k_2 b)} \right] \quad (56)$$

and

$$\Xi = \left[\left(\frac{m\pi}{a} \right)^2 + k_a^2 \right]^2 - k_p^4 \quad (57)$$

Similarly, a modal, transfer mobility function, defined by the ratio of the edge rotational velocity per unit incident pressure, can be derived from the above solution for the response. If the subscript 2 represents the location of the junction, the modal transfer mobility for mode m is given by,

$$M_{21_m} = \frac{j\omega}{P_0} \frac{\partial u_1(y=b)}{\partial y} = \frac{\omega 4\Gamma_1}{aD^*} \frac{k_a e^{-jk_a b}}{\Xi} + j\omega \left[A'k_1 \sinh(k_1 b) + B'k_1 \cosh(k_1 b) \right. \\ \left. - C'k_2 \sin(k_2 b) + D'k_2 \cos(k_2 b) \right] + j\omega K_1 k_2 \cos(k_2 b) \quad (58)$$

The input mobility is a function of both the mode number and the variable y while the transfer mobility is a function of only the mode number. The reason for the y dependency for the input mobility is that the excitation is distributed over the surface of the plate structure and thus this input mobility represents the response per unit incident pressure anywhere on the surface of the plate structure.

3.5. SIMPLY SUPPORTED PLATE WITH AN EDGE MOMENT

An approach similar to the one used for the plate subsystem discussed in the previous section is used for this second plate subsystem with different loading conditions. In this case no incident acoustic waves are considered, that is $p_i(x,y,z) = 0$. The forces acting on this plate subsystem are the edge moment and the scattered pressure. The same equation of motion, as equation (27), applies with the pressure term on the right hand side only representing the scattered pressure. Because of the presence of the edge moment, the boundary conditions for the edge $y=b$ of this plate subsystem are different from those given in equation (36). The boundary conditions for the edge $y=b$, are given by,

$$u(y=b) = 0 ; \quad \frac{\partial^2 u(y=b)}{\partial y^2} = - \frac{T_m}{D_p} \quad (59)$$

where T_m represents the mode m component of the edge moment [2]. The plate response is obtained by solving the equation of motion as given by equation (27), with $p(x,y,z=0)$ replaced by the scattered pressure component. In obtaining a solution to this equation of motion, a first approximation is obtained by neglecting the scattered pressure, in which case it becomes a homogeneous equation of motion. The solution that satisfies the boundary conditions is given by,

$$u_h(y) = u(y) = \frac{T_m}{2\sqrt{\rho D^*} \cdot \omega} \left[\frac{\sin(k_2 y)}{\sin(k_2 b)} - \frac{\sinh(k_1 y)}{\sinh(k_1 b)} \right] \quad (60)$$

To account for the presence of the scattered pressure component, a corrective factor K_{II} is introduced

$$u_1(y) = \frac{T_m}{2\sqrt{\rho D^*} \cdot \omega} \left[K_{II} \frac{\sin(k_2 y)}{\sin(k_2 b)} - \frac{\sinh(k_1 y)}{\sinh(k_1 b)} \right] \quad (61)$$

$u_1(y)$ represents the exact solution of the equation of motion including the scattered pressure. The error introduced by the approximation is again given by an equation of the form of equation (40) where in this case as well $u_e(y)$ must satisfy equation (41). However, the boundary conditions that apply to the term $u_e(y)$ in this case are different from those given in equation (42). The boundary conditions are modified to include the influence of the edge moment.

$$u_e(y=0 \text{ or } b) = -u_1(y=0 \text{ or } b) = -K_{II} u(0 \text{ or } b) = 0$$

$$\begin{aligned} \frac{\partial u_e^2(y=0)}{\partial y^2} &= - \frac{\partial u_1^2(y=0)}{\partial y^2} \\ \frac{\partial u_e^2(y=b)}{\partial y^2} &= - \frac{\partial u_1^2(y=b)}{\partial y^2} - \frac{T_m}{D^*} \end{aligned} \quad (62)$$

The solution for $u_e(y)$ is derived in terms of the Green's function (similar equations to (43) and (44)). The application of the boundary conditions into equation (43), and setting $u_e(y) = 0$ for

all values of y leads to an equation involving $p_s(y)$. After some substitutions and manipulation of the terms, a correcting factor term K_{II} is obtained as follows,

$$K_{II} = \frac{\frac{k_2^2 \cos(k_2 b)}{k_1^2 + k_2^2} + \frac{j}{2\pi^2 a} \left[\frac{m\pi}{a} \right]^2 \alpha_1 \int_{-\infty}^{\infty} u_2(\beta) I(\beta) S(\beta) d\beta}{-\frac{j}{2\pi^2 a} \left[\frac{m\pi}{a} \right]^2 \alpha_1 \int_{-\infty}^{\infty} u_1(\beta) I(\beta) S(\beta) d\beta} \quad (63)$$

where

$$u_1(\beta) = \frac{\sin[k_2(y-b)]}{k_1^2 + k_2^2} \quad (64)$$

and

$$u_2(\beta) = \frac{\sinh[k_1(y-b)]}{[k_1^2 + k_2^2] \sinh(k_1 b)} \quad (65)$$

From the solution for the response of the plate subsystem subjected to an edge moment and including the influence of the fluid loading, an input mobility function is obtained for the edge of the plate. Defining the edge input mobility as the rotational velocity response per unit applied edge moment,

$$\begin{aligned} M_{2m} &= \frac{j\omega}{T_m} \frac{\partial u(y=b)}{\partial y} \\ &= \frac{j}{2\sqrt{\rho D^*}} \left[K_{II} k_2 \cos(k_2 b) - \frac{k_1}{\tanh(k_1 b)} \right] \end{aligned} \quad (66)$$

If the two plates of the L-shaped plate structure are identical then,

$$M_{2m} = M_{3m} \quad (67)$$

The subscript 3 represents the connected edge of the receiver plate [2].

The transfer mobility function for the plate surface velocity response per unit applied edge moment can also be defined from the above analysis,

$$M_{12_m} = \frac{j}{2\sqrt{\rho D^*}} \left[K_{II} \sin(k_2 y) - \frac{\sinh(k_1 y)}{\sinh(k_1 b)} \right] \quad (68)$$

In this case as well, the input mobility functions are only a function of the mode number m . However, the transfer mobility is a function of both the variable y and the mode number m , since it represents a transfer to any point on the surface of the plate.

3.6. INPUT AND TRANSFER POWER EQUATIONS

Having derived the mobility functions, the derivation of the power flow equations follows in the same way as for the distributed mechanical excitation. The input power is obtained from both an integral over the spatial variable (y) and a summation over all the modes (m). The modal summation is an alternative way of performing a spatial integration when the response can be decomposed into a set of modes.

For the transferred power, this is given by an integral along the length of the junction or alternatively since this is the direction for which a modal decomposition has been assumed, the total transferred power is given by a summation over all the modes. Since the transferred power is dependent on the edge moment which is controlled by the incident acoustic excitation, to evaluate the edge moment and hence the transferred power an integral still has to be performed for the y direction, the direction perpendicular to the junction.

The total input power is given by the product of the total pressure acting on the source plate surface and the plate velocity response integrated over the y direction and summed for all modes m .

$$P_{\text{input}} = \frac{a}{4} \sum_{m=1}^{\infty} \text{Real} \int_0^b \left[2p_{i_m}(y) + p_{s_1}(y) + p_{s_2}(y) \right] V_1^*(y) dy \quad (69)$$

where the two p_s terms represent the scattered pressure components associated one with the response of the source plate due to the incident sound wave and one due to the application of the edge moment. V_1^* is the surface velocity of the source plate for mode m and is given by,

$$V_1(y) = P_o M_{1_m}(y) + T_m M_{12_m}(y) \quad (70)$$

Eliminating T_m by solving for continuity of motion at the junction edge,

$$T_m M_{3m} = T_m M_{2m} = -T_m M_{2m} + P_o M_{21m}$$

$$T_m = \frac{P_o M_{21m}}{2 M_{2m}} \quad (71)$$

substituting into equation (70),

$$V_1(y) = P_o \left[M_{1m}(y) + \frac{M_{21m}}{2M_{2m}} M_{12m}(y) \right] \quad (72)$$

Substituting into the power input expressions the values for $V_1(y)$ and for the scattered pressure components the following result is obtained for the input power;

$$P_{input} = \frac{a}{4} \sum_{m=1}^{\infty} \left\{ \text{Real} \left[\int_0^b 2p_i(y) P_o^* \left[M_{1m}(y) + \frac{M_{21m}}{2M_{2m}} M_{12m}(y) \right] dy \right. \right.$$

$$\left. \left. + P_o^2 \frac{\omega \rho_o}{4\pi^2} \left(\frac{m\pi}{a} \right)^2 \text{Real} \left[\int_{-\infty}^{\infty} I(\beta) \left| M_{1m}(\beta) + \frac{M_{21m}}{2M_{2m}} M_{12m}(\beta) \right|^2 d\beta \right] \right\} \quad (73)$$

The last integral of the above equation the variable y has been changed to the variable β which represents the fourier transform variable with respect to y . Also, this last integral represents the power flow out of the source plate, as radiated acoustical power from one side of the source plate.

The transferred power between the two plates is given by the summation for all the mode contributions for the product between the edge moment and the edge rotational velocity. This can also be expressed in terms of the modal mobility functions derived above,

$$P_{trans} = \frac{a}{4} \sum_{m=1}^{\infty} P_o^2 \left| \frac{M_{21m}}{2M_{2m}} \right|^2 \text{Real} \left(M_{2m} \right) \quad (74)$$

3.7. RESULTS

Results for the power input, power transfer and radiated acoustical power are obtained using the above power flow expressions for the structure shown in figure (1). Acoustic waves are considered incident on the source plate such that both θ and ϕ are equal to 45° . The results for the input and transferred power for this angle of incidence are shown in figure (6). Figure (7) shows the acoustic power radiated by the source plate obtained from the scattered pressure component. As can be observed from this figure the contribution to the scattered pressure from motion of the source plate due to the presence of the edge moment is not significant.

To determine the influence of the fluid loading on the power flow, the input and transferred power are computed when the fluid loading effects (the scattered pressure and hence the coefficients K) are neglected. Figures (8) and (9) show a comparison between the two sets of results. As can be observed from these figures the influence of the fluid loading is not significant, although some differences are observed mainly near the resonant frequencies and in the trough between the resonant frequencies. The main influence of the fluid loading is to increase the damping of the structure due to the acoustic radiation.

For other angles of incidence, $\theta = 0, 15, 75$ degrees and $\phi = 0, 30$ and 60 degrees, power input and power transfer curves are shown in figures (10), (11) and (12). The number of modes in the results shown in figure (10) is reduced as compared to the other results mainly because of the symmetry of the excitation. The even modes are not excited with normal incidence. Apart from the number of modes that are excited the general shape of the power flow curves are also influenced by the angles of incidence.

4. ACOUSTIC INTERACTION

The scattered pressure generated by each of the coupled plate subsystems will influence the response of the other subsystems. The approach that can be used to consider this interaction is described in this section. Assuming that the plate insonified by the incident acoustic waves is considered is surrounded by a rigid baffle. On the opposite side of the excitation, in the cavity between the two plates, acoustic waves are generated due to the response of the two plates (figure 13) which will influence the response of each of the subsystems. The acoustic field inside this cavity can be obtained by considering the scattered pressures generated by the each of the two plate subsystems, taking into account the change in the boundary conditions due to the presence of the second plate.

If the acoustic interaction between the plates is considered, the different components of the L-shaped plate can be considered to be equivalent to the three set-ups shown in figure (14). The first set-up consists of a simply supported plate surrounded by a baffle, with an acoustic wave incidence on the plate surface on one side, and with a second baffle perpendicular to the plate located along one of the plate edges, the edge that forms the junction with the second plate, (figure 14(a)). The second set-up is similar to the first set-up except for the presence of the external incident acoustic wave excitation. In this case the excitation is from an edge moment (figure 14(b)) The third set-up is similar to the second set-up except for the orientation of the baffles, (figure 14(c)). Each of these set-ups must be separately considered to obtain the required set of mobility functions.

To deal with the presence of the baffles, these can be substituted by by equivalent image plates as shown in figure (15), where it is assumed that the plates' structural characteristics are identical, that is both plates have the same size, thickness and damping. Since for an incident acoustic wave on a rigid surface, the reflected wave suffers no change in phase, the amplitude of motion of the image plate surface is the same as that of the real plate surface. If the origin is placed at the location of the junction between the real plate and its image, and if $u(x,y)$ represents the transverse displacement of the plate, where y is the coordinate perpendicular to the baffle, then,

$$u(x, -y) = u(x, y) \quad (75)$$

$u(x, -y)$ represents the displacement of the image plate and $u(x, y)$ represents the displacement of the real plate. The presence of the image plate will modify the scattered pressure component. Because the interaction between the two plates is limited to one side of the L-shaped plate configuration, the scattered pressure on either side of the first set-up is not symmetrical. Same applies for the case of the edge moment excitation (second set-up). For the third set-up, the arrangement is symmetrical, and therefore the same scattered pressure applies on both sides of the plate. A condition which can be assumed for this third set-up is that the plate is surrounded by a rigid baffle. This is not necessary in the development of the solution, free boundary conditions or other boundary conditions can be assumed as is done in reference [10].

The response of the system shown in figure (15(a)), with all four edges simply supported and with an acoustic wave obliquely incident on one side of the plate can be obtained from a solution to the wave equation of motion describing the response of the plate, similar to the approach used in the previous section. The excitation pressure can in this case as well be written in the form of equation (25).

$$p(x, y, z) = 2 p_i(x, y, z) + p_{s1}(x, y, z_+) + p_{s2}(x, y, z_-) \quad (76)$$

where p_i is the incident pressure, including both the external acoustic field and the pressure created by the second plate, p_{s1} is the scattered pressure component acting on the z_+ side and p_{s2} the scattered pressure component acting on the z_- side, (figure (15(a))).

The solution to the wave equation can proceed in the same way as in previous section, with the appropriate changes in the term $u(\beta)$ which forms part of the expression for the scattered pressure. The term is the spatial transform of the response of the plate in the direction perpendicular to the edge which will be connected to the second plate. For the present case, this term has two components, since the scattered pressure is considered on both sides of the plate. However, the two components are not identical since a reflecting baffle perpendicular to the plan of the plate is located on the one side opposite the excitation. Thus, using equation (A.22),

$$\frac{p_{s_t}(\xi)}{D^*} = \frac{j}{2\pi^2 a} \left[\frac{m\pi}{a} \right]^2 \alpha_1 \int_{-\infty}^{\infty} I(\beta) u_t(\beta) e^{-j\beta\xi} d\beta \quad (77)$$

where

$$p_{s_t}(y) = p_{s1}(y) - p_{s2}(y) \quad (78)$$

and

$$u_t(\beta) = u_1(\beta) + u_2(\beta) \quad (79)$$

$$u_1(\beta) = \int_0^b u(y) e^{j\beta y} dy \quad (80)$$

$$u_2(\beta) = 2 \int_0^b u(y) \cos(\beta y) dy \quad (81)$$

The solution to the response of the plate is, from this point onwards, identical to that in the previous section.

For the case in figure (15(b)), the plate is subjected to the same form of excitation as the system in figure (15(a)) with the additional excitation by an edge moment. An incident pressure on the inside surface of the plate is considered to 'account' for the acoustic field generated by the other plate when coupled. This pressure will be independent of the response of this plate. For the scattered pressure components, these are identical to those given by equations (77) and (78).

For the third set-up, the excitation is from the edge moment, and on one side from an external acoustic pressure representing the acoustic field generated by the other plate when coupled. The scattered pressure components on both sides of the plate are identical because of the symmetry in the location of the baffles.

For this system, the expressions for $u(\beta)$ for both sides of the plate are given by equations similar to equation (81). Having solved for the response of each of the plate subsystems, mobility functions can be derived and used in the MPF model in the same way as was done previously.

The input and transferred power to the source and receiver plate respectively can be obtained in the same way as in section (3.6).

5. CONCLUSION

The mobility power flow approach for two coupled plate structures with excitation from distributed loading, including acoustic excitation and the interaction from the scattered acoustic pressure has been presented. For a distributed loading which is independent of the motion of the structure, the mobility power flow approach is relatively straight forward and very much similar to that of point loading [2].

The extension of the Power Flow Method to excitation conditions other than mechanical excitation has been demonstrated in this report. The excitation considered here is an incident acoustic wave. In this case of acoustic excitation, the response of the structure influences the incident acoustic field and the problem of the structure response becomes a fluid-structure interaction problem.

Since most of the work found in the literature on fluid-structure interaction deals with simply supported plate structures, because of the requirement of an apriori knowledge of the vibration mode shape, which is not valid in the case of connected plate structures, an approximate solution based on the work by Leppington [8] has been used here. Although fluid loading is considered, it is found that light fluid loading does not significantly modify the mode shape of the vibrating plate structure but that the scattered pressure can be significant.

If the results obtained here for the power input and transferred for the L-shaped plate acoustic excitation are compared to results for mechanical excitation [6], the following observation can be made. With uniformly distributed mechanical excitation, the power flow is similar to that for excitation from normal incidence acoustic waves. For oblique incidence waves additional modes of vibration are excited same as in the case of point excitation. Compared with the power flow results for point excitation, the power flow is higher in the case of the mechanical excitation. For these comparisons the total load on the source plate is kept constant.

Presented in this paper is a MPF analysis of the interaction of the acoustic waves generated by two coupled plate substructures. The analysis can be extended to other coupled plate substructures to enclose an acoustic space. While, the influence of the acoustic interaction is not so significant when dealing with interactions through an open acoustic space, if the acoustic medium is enclosed by components of the structure, standing waves can be set up within the acoustic medium resulting in a strong acoustic field, which can interact strongly with the response of elements of the structure.

As a final note, an important result of this report is that the effects of fluid loading on a connected plate structure can be integrated into the general Power Flow Method. This enhances the usefulness of power flow methods in determining the response of plate-like structural components and their interaction with the surrounding medium.

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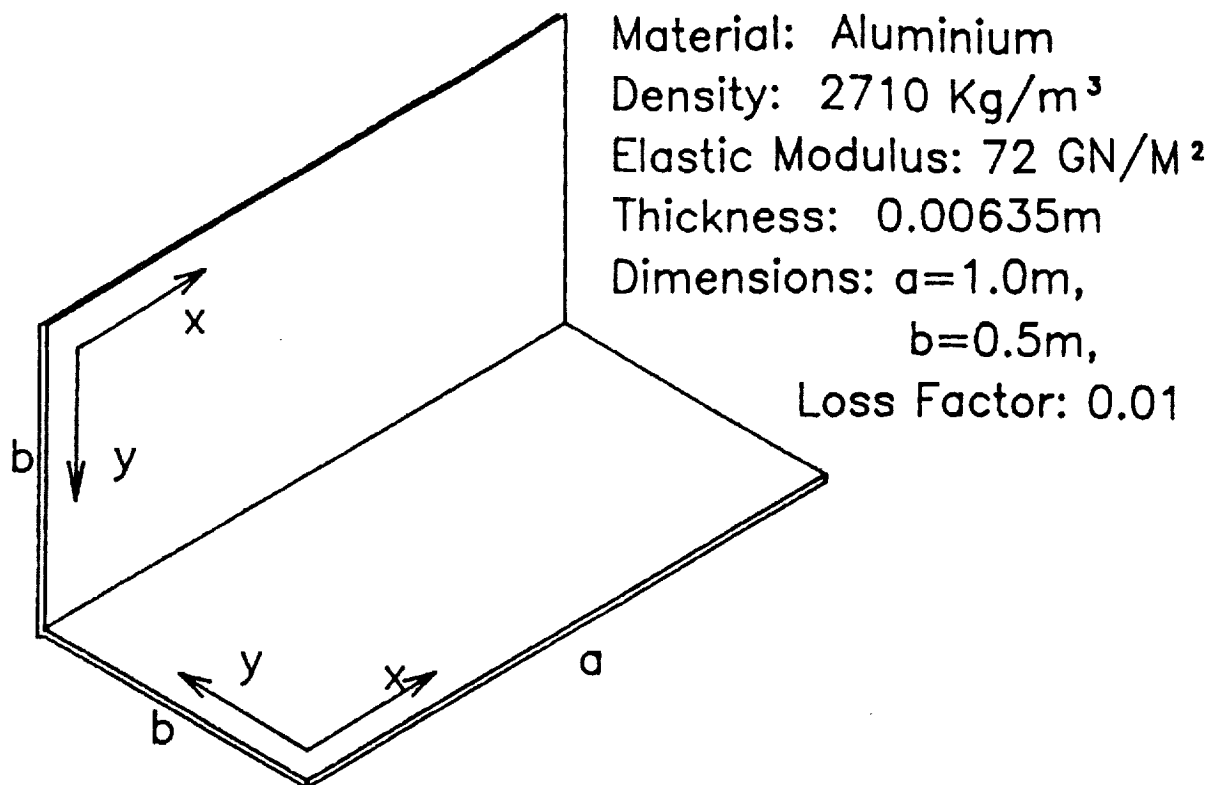


Figure 1. L-shaped plate structure, giving plates' characteristics and orientation of axis.

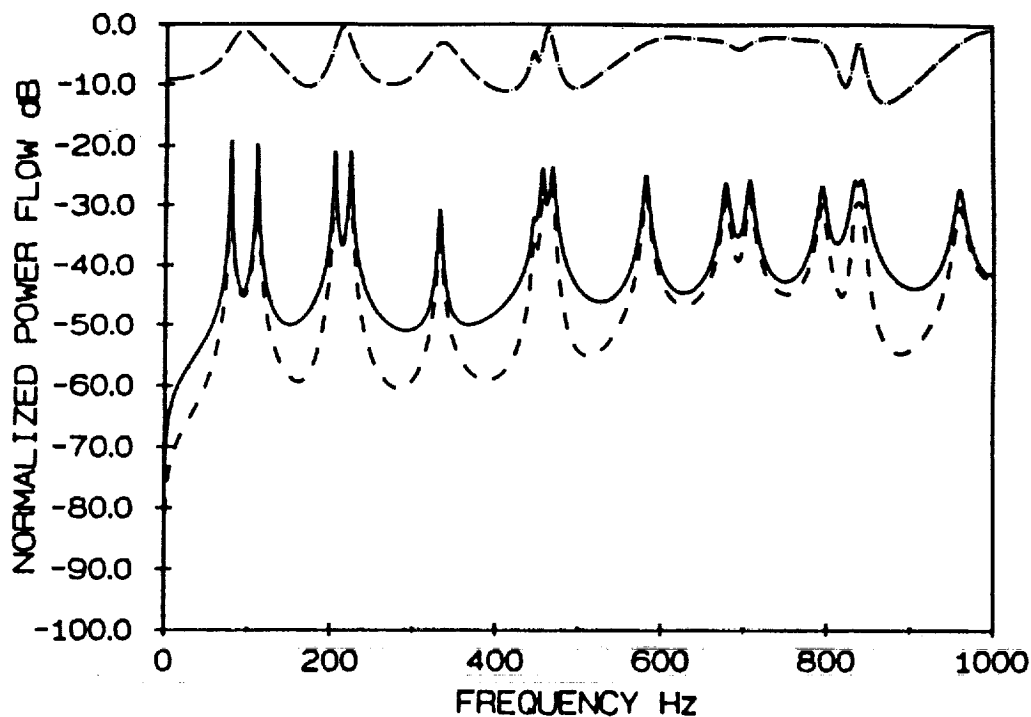


Figure 2. Normalized power flow results for point load excitation at center of source plate. —: Power input; ---: power transfer; -.-.-: power ratio.

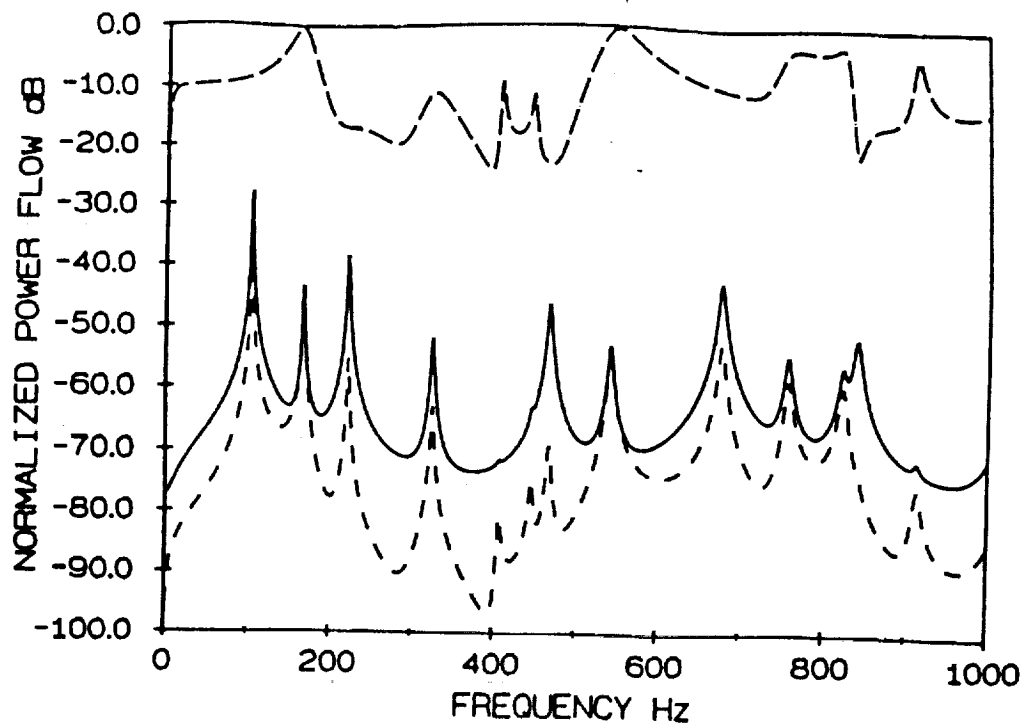


Figure 3. Normalized power flow results for uniform distributed load excitation. —: Power input; ---: power transfer; - · - · -: power ratio.

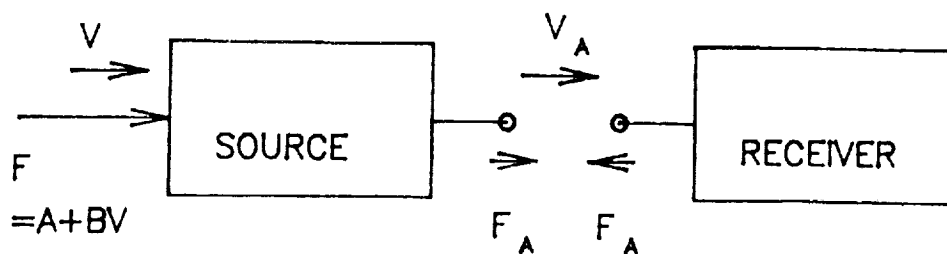


Figure 4. Source receiver model with the input load dependent on the response of the source structure.

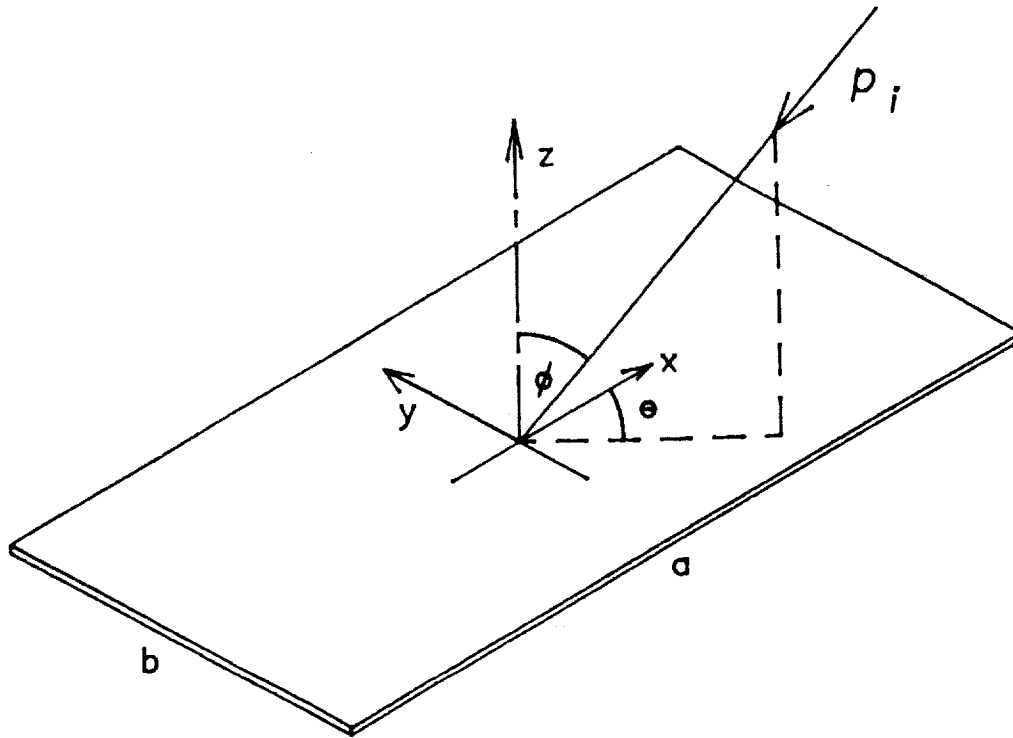


Figure 5. Incident acoustic waves at oblique angles ϕ and θ .

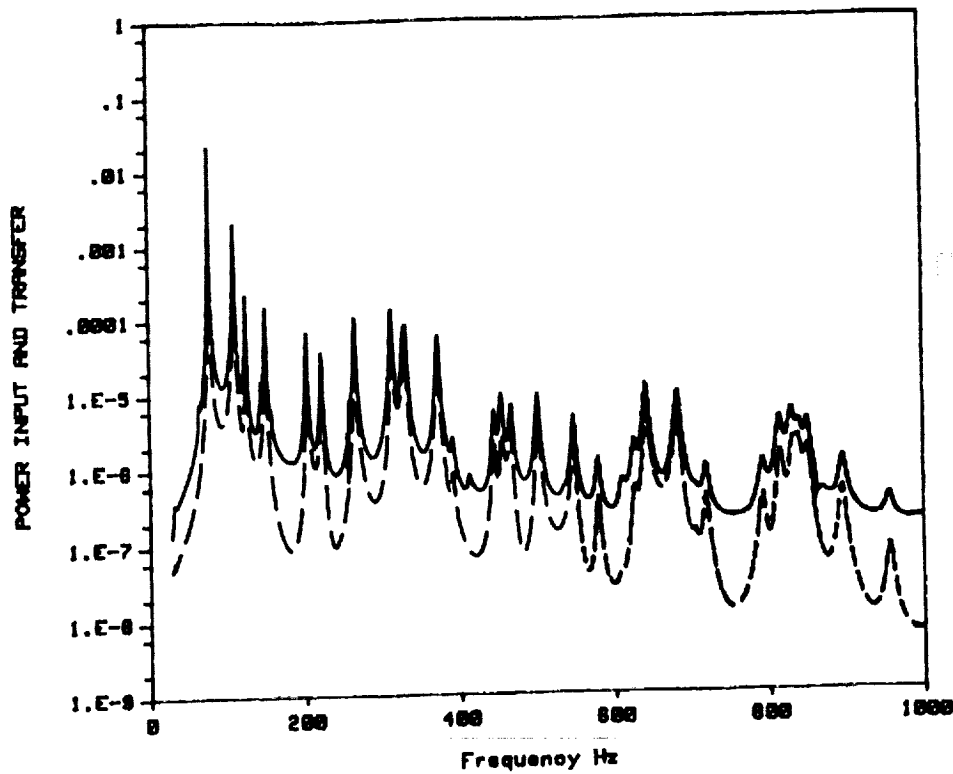


Figure 6. Normalized input and transfer power per unit incident pressure, including the influence of fluid loading. Incidence angles $\phi = 45^\circ$ and $\theta = 45^\circ$. —: Power input; ---: power transfer.

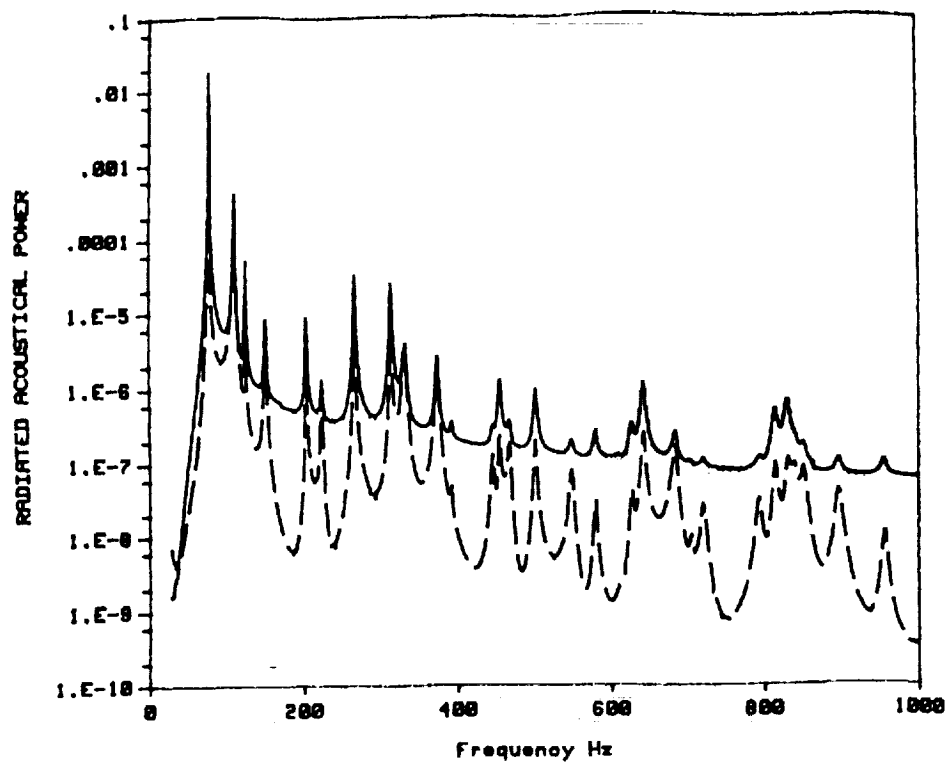


Figure 7. Radiated acoustical power by source plate per unit incident acoustic pressure. —: Total radiated power; ---: radiated power contribution from edge moment.

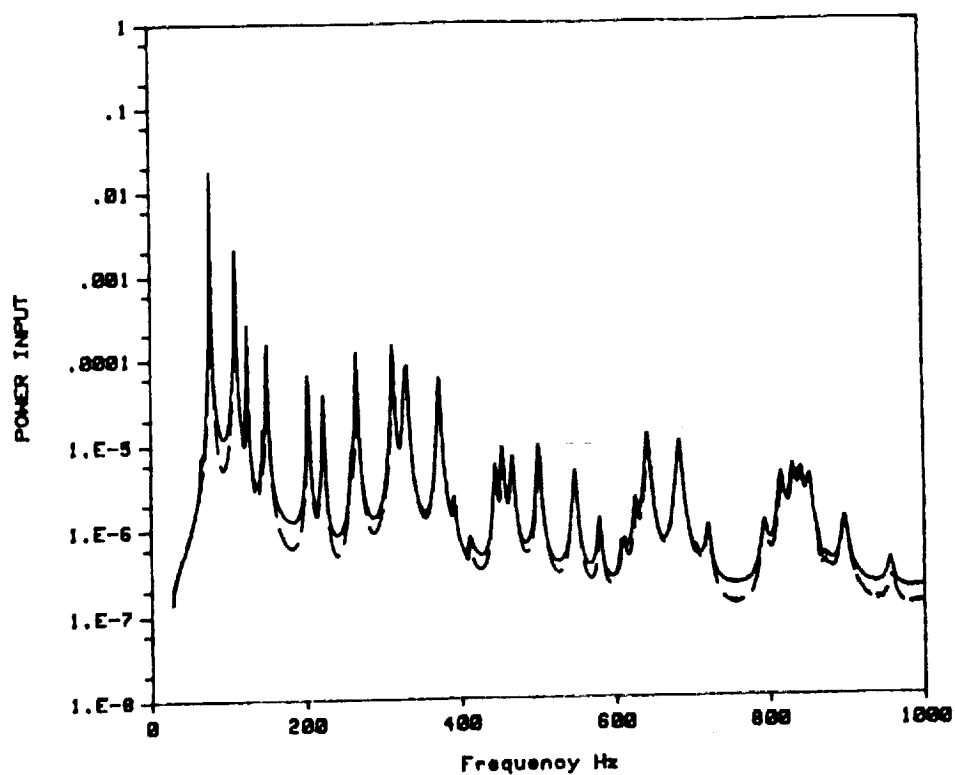


Figure 8. Normalized input power per unit incident pressure, with and without the influence of fluid loading. —: With fluid loading; ---: without fluid loading.

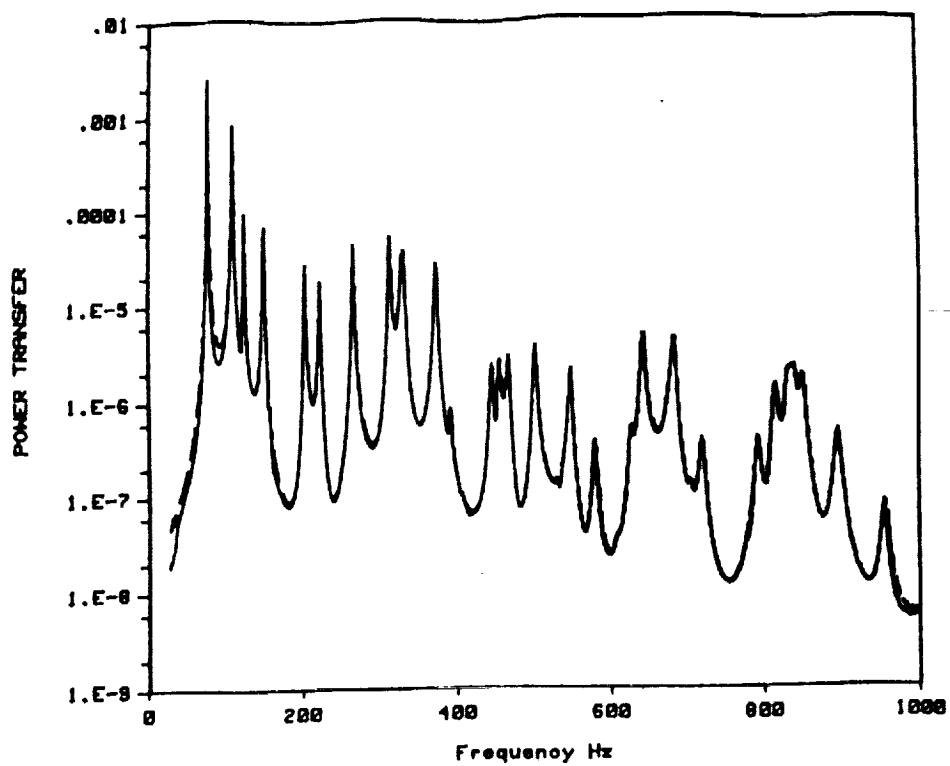


Figure 9. Transferred power per unit incident pressure, with and without the influence of fluid loading. —: With fluid loading; ---: without fluid loading.

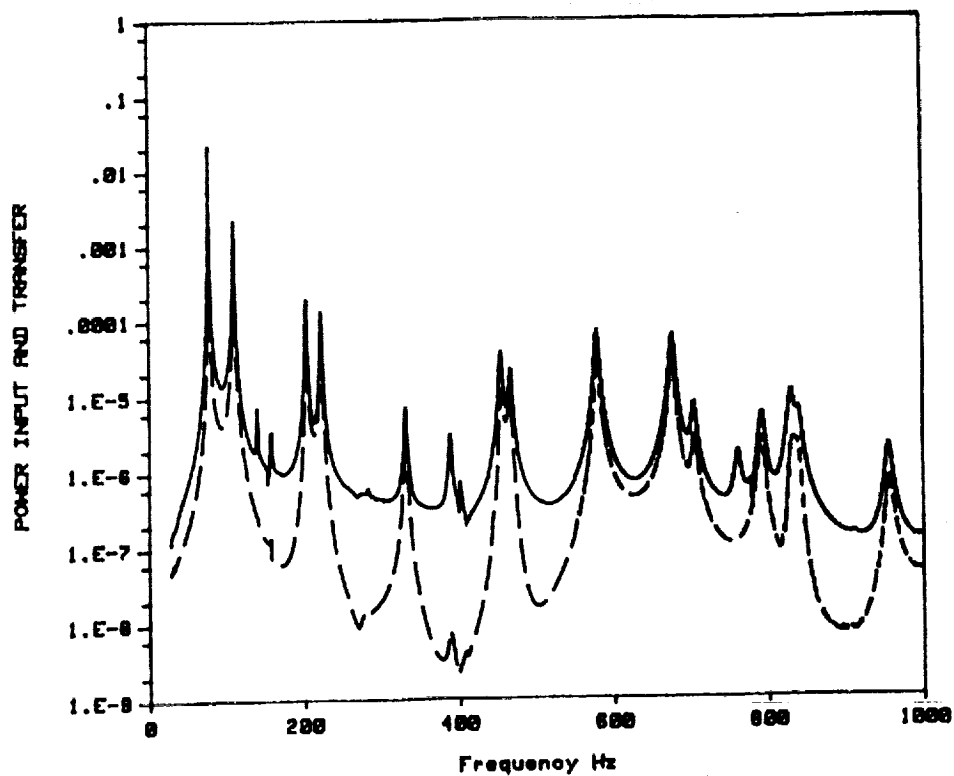


Figure 10. Power flow per unit incident pressure with fluid loading. Incidence angles $\phi = 0^\circ$ and $\theta = 0^\circ$. —: Power input; ---: power transfer.

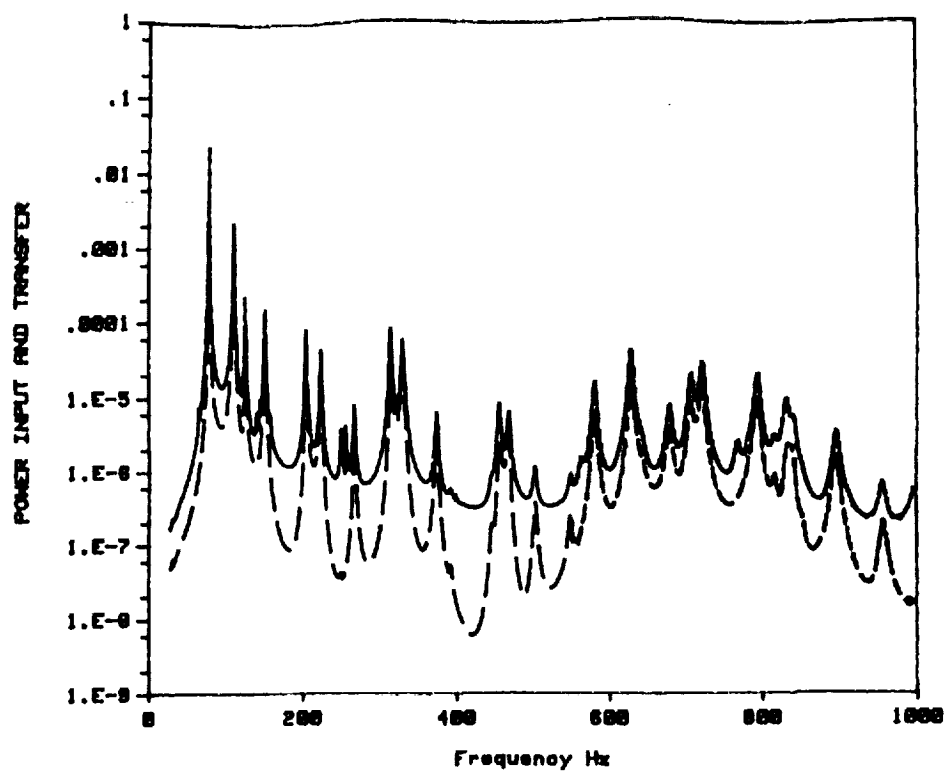


Figure 11. Power flow per unit incident pressure with fluid loading. Incidence angles $\phi = 0^\circ$ and $\theta = 15^\circ$. —: Power input; ---: power transfer.

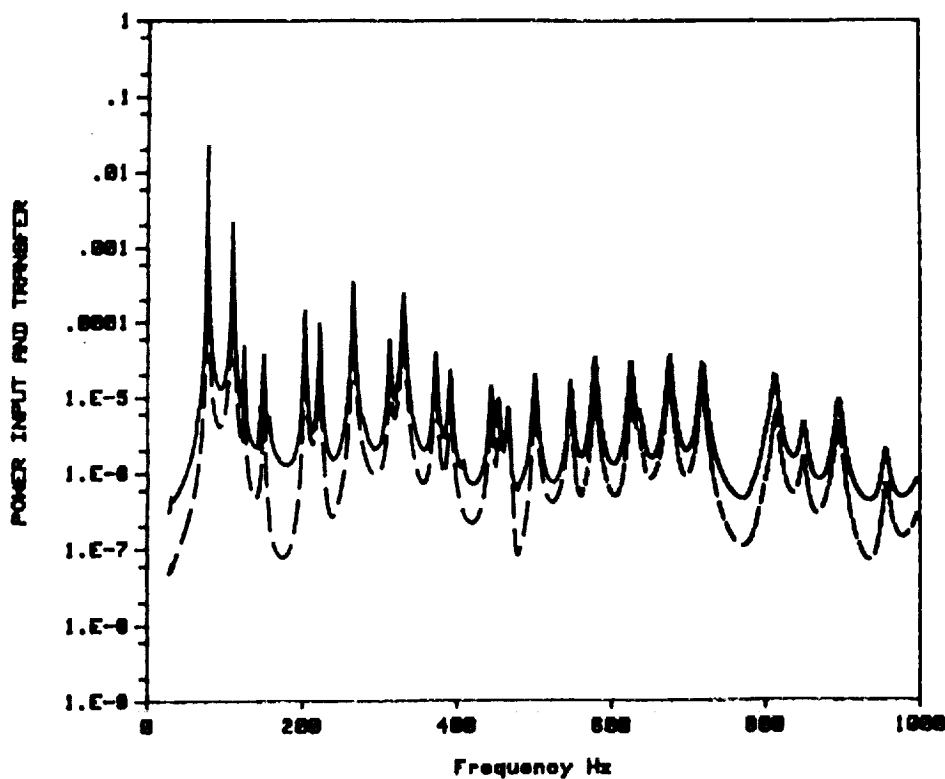


Figure 12. Power flow per unit incident pressure with fluid loading. Incidence angles $\phi = 75^\circ$ and $\theta = 75^\circ$. —: Power input; ---: power transfer.

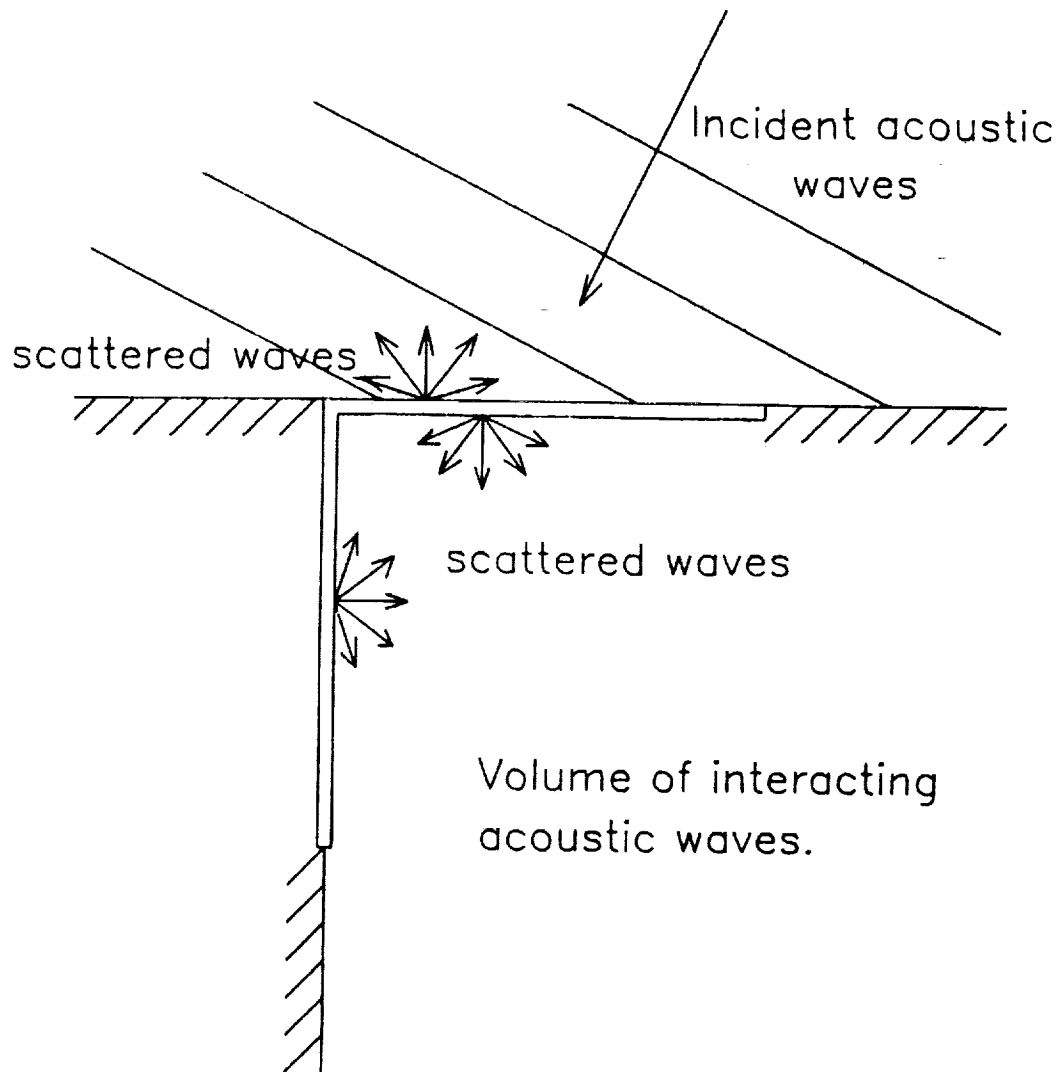


Figure 13. Acoustic interaction model.

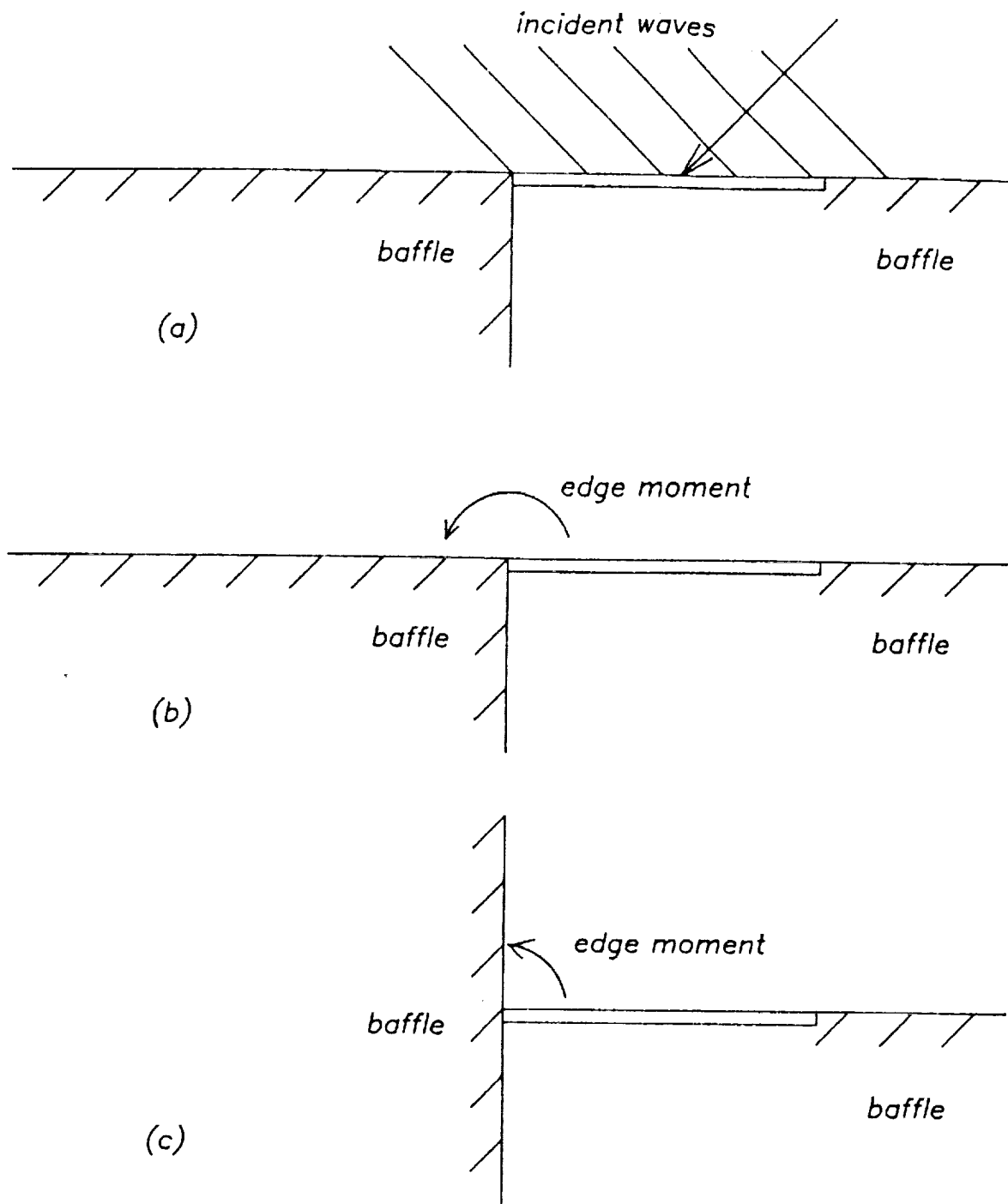


Figure 14. MPF subsystem models for L-shaped plate model including acoustic interaction.

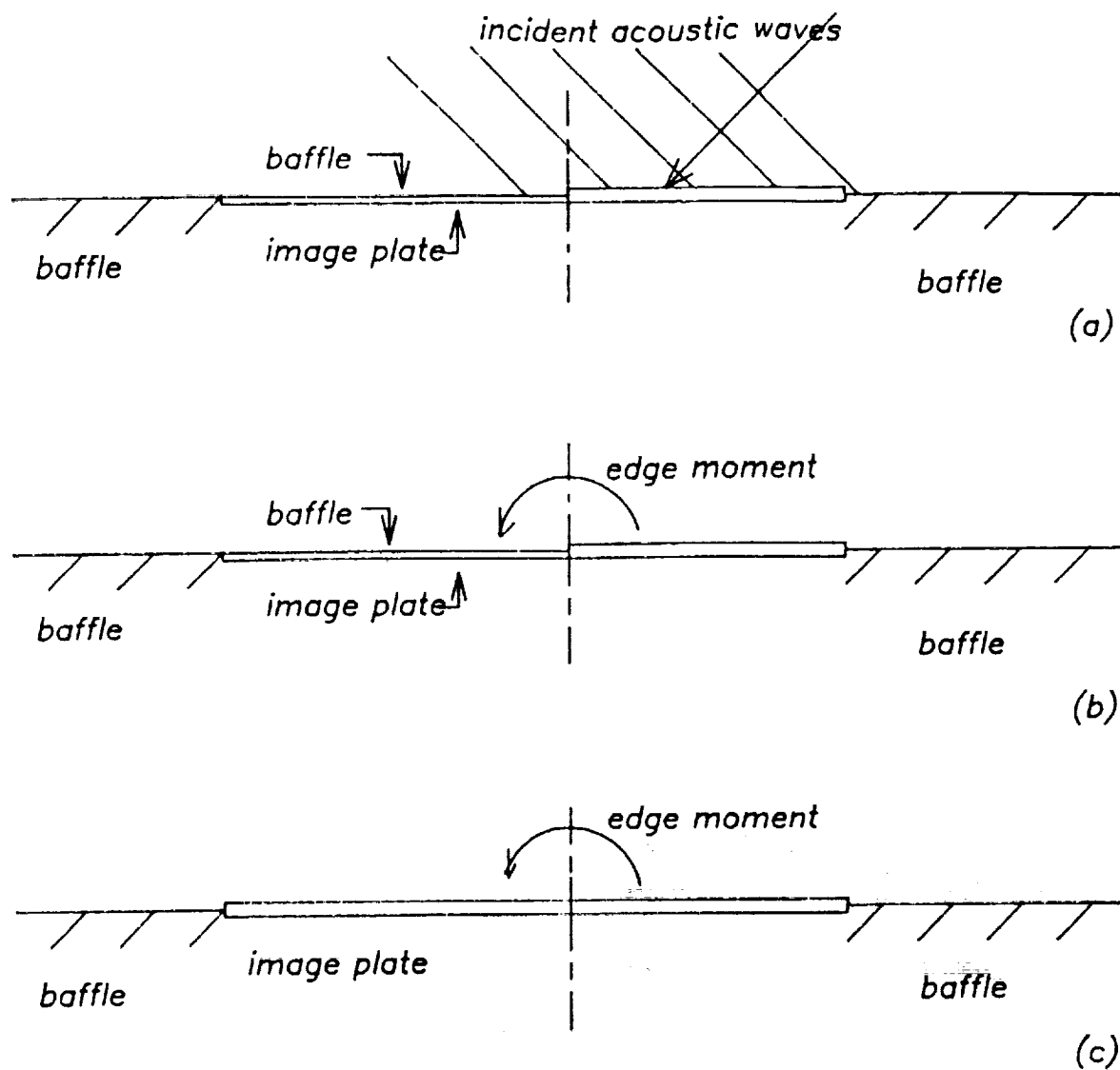


Figure 15. Equivalent subsystems for coupled plates including the acoustic coupling between the subsystems.

APPENDIX

RELATIONSHIP BETWEEN SCATTERED PRESSURE AND NORMAL SURFACE VELOCITY

In deriving the relationship between the scattered pressure and the normal surface velocity, the following conditions are assumed:

(a) the plate normal displacement has a harmonic time dependency of the form,

$$u(x, y, t) = u(x, y) e^{j\omega t} \quad (A.1)$$

(b) the plate is finite and lies in the plane $(x, y, z=0)$, with the acoustic fluid occupying the half-space $z \geq 0$;

(c) the propagation of a plane acoustic wave in three dimensional space can be described by:

$$p(x, y, z, t) = P_0 \exp(-jk_x x - jk_y y - jk_z z) e^{j\omega t} \quad (A.2)$$

Defining a Fourier transform of $u(y)$ by:

$$u(\beta) = \int_{-\infty}^{\infty} u(y) \exp(j\beta y) dy \quad (A.3)$$

and the inverse transform by:

$$u(y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} u(\beta) \exp(-j\beta y) d\beta \quad (A.4)$$

From the momentum equation in the z direction:

$$\left. \frac{\partial p}{\partial z} \right|_{z=0} + \rho_0 \frac{\partial V}{\partial t} = 0 \quad (A.5)$$

where p is the acoustic pressure, ρ_0 the fluid mean density and V the particle velocity in the z direction. From equation (A.5),

$$-jk_z p_s(x, y, z) \exp(j\omega t) = -j\omega \rho_0 V(x, y, z) \exp(j\omega t) \quad (A.6)$$

where p_s is the scattered pressure.

At the interface between the plate and the acoustic medium ($z=0$), the surface normal velocity is equal to the acoustic particle velocity. Therefore, at the interface,

$$p_s(x, y) = \frac{\omega p_o}{k_z} v(x, y) \quad (\text{A.7})$$

where k_z must satisfy the condition:

$$k_x^2 + k_y^2 + k_z^2 = \left(\frac{\omega}{c} \right)^2 = k^2 \quad (\text{A.8})$$

where k is the acoustic wavenumber, ω is the radian frequency and c the acoustic wave speed.

Let α and β be the plate wavenumbers in the x and y directions respectively. Since the acoustic wave is generated by the plate motion, the x and y variations of the acoustic field must follow those of the plate and therefore:

$$k_x = \alpha \quad \text{and} \quad k_y = \beta \quad (\text{A.9})$$

that is,

$$k_z^2 = k^2 - \alpha^2 - \beta^2 = k^2 - k_p^2 \quad (\text{A.10})$$

where k_p is the plate bending wavenumber.

Spatial Fourier transforming equation (A.7) in the x and y directions.

$$p_s(\alpha, \beta) = \frac{\omega p_o}{k_z} v(\alpha, \beta) \quad (\text{A.11})$$

For a finite rectangular plate, simply supported at $x = 0$ and $x = a$, the response of the plate can be described by:

$$v(x, y) = \sum_{p=1}^{\infty} v_p(y) \sin \left[\frac{p\pi x}{a} \right] \quad (\text{A.12})$$

Fourier transforming this expression for $V(x, y)$,

$$v(\alpha, \beta) = \sum_{p=1}^{\infty} v_p(\beta) \left[\frac{p\pi}{a} \right] \frac{(-1)^p e^{j\alpha a} - 1}{\alpha^2 - \left[\frac{p\pi}{a} \right]^2} \quad (\text{A.13})$$

The scattered pressure component, $p_s(x, y)$, can also be written in the form,

$$p_s(x, y) = \sum_{m=1}^{\infty} p_{s_m}(y) \sin\left[\frac{m\pi x}{a}\right] \quad (\text{A.14})$$

where

$$p_{s_m}(y) = \frac{2}{a} \int_0^a p_s(x, y) \sin\left[\frac{m\pi x}{a}\right] dx$$

Fourier transforming in the y direction,

$$p_{s_m}(\beta) = \frac{2}{a} \int_0^a p_s(x, \beta) \sin\left[\frac{m\pi x}{a}\right] dx \quad (\text{A.15})$$

Fourier transforming also in the x direction,

$$p_{s_m}(\beta) = \frac{2}{a} \int_0^a \left[\frac{1}{2\pi} \int_{-\infty}^{+\infty} p_s(\alpha, \beta) e^{-j\alpha x} d\alpha \right] \sin\left[\frac{m\pi x}{a}\right] dx \quad (\text{A.16})$$

$$= \frac{1}{\pi a} \int_{-\infty}^{\infty} p_s(\alpha, \beta) \left[\frac{m\pi}{a} \right] \frac{(-1)^m e^{-j\alpha a} - 1}{\alpha^2 - \left[\frac{m\pi}{a} \right]^2} d\alpha \quad \text{A.17}$$

Combining equation (A.11), (A.13) and (A.17) yields,

$$p_{s_m}(\beta) = \frac{\omega p_o}{\pi a} \left[\frac{m\pi}{a} \right] \sum_{p=1}^{\infty} \left\{ \left[\frac{p\pi}{a} \right] v_p(\beta) \right. \\ \left. \int_{-\infty}^{+\infty} \frac{(-1)^m e^{j\alpha a} - 1}{\alpha^2 - \left[\frac{m\pi}{a} \right]^2} \frac{(-1)^p e^{j\alpha a} - 1}{\alpha^2 - \left[\frac{p\pi}{a} \right]^2} \frac{d\alpha}{k_z} \right\} \quad (\text{A.18})$$

The terms of the summation for which $p \neq m$ represent intramodal coupling which has been shown to be negligible for light fluid loading provided the modal density of the structure is low [7].

Therefore equation (A.18) can be simplified to:

$$p_m(\beta) = \frac{\omega \rho_o}{\pi a} \left[\frac{m\pi}{a} \right]^2 I_m(\beta) V_m(\beta) \quad (\text{A.19})$$

where

$$I_m(\beta) = 2 \int_{-\infty}^{+\infty} \frac{[1 - (-1)^m \cos(\alpha a)]}{[\alpha^2 - \left(\frac{m\pi}{a}\right)^2]^2} \frac{d\alpha}{k_z} \quad (\text{A.20})$$

From equation (A.19), equation (44) can be derived from the definition of the inverse transform,

$$p_s(\xi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} p_{sm}(\beta) \exp(-j\beta\xi) d\beta \quad (\text{A.21})$$

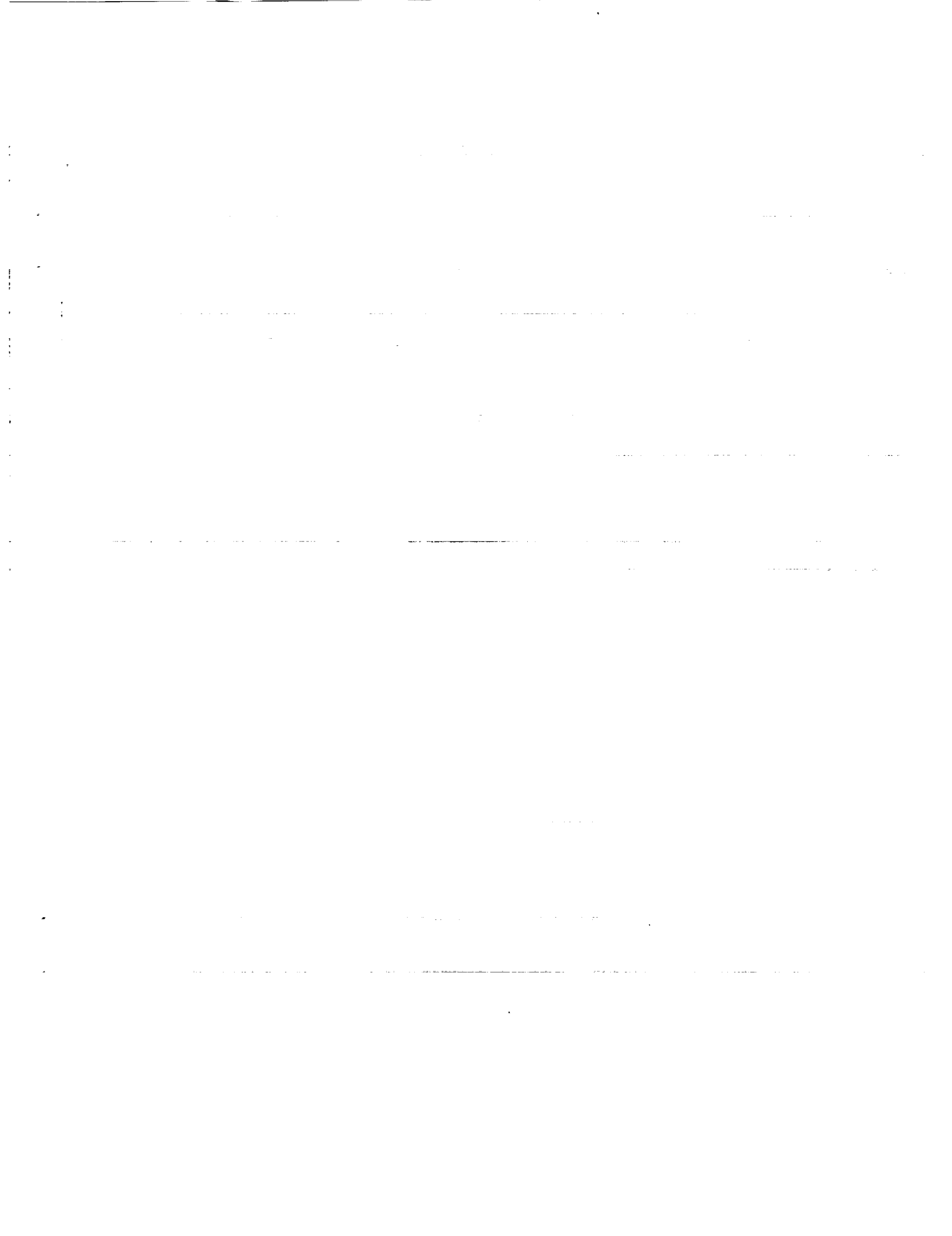
and substituting for $p_{sm}(\beta)$,

$$p_{sm}(\xi) = \frac{1}{2\pi} \frac{\omega \rho_o}{\pi a} \left[\frac{m\pi}{a} \right]^2 \int_{-\infty}^{+\infty} I_m(\beta) V_m(\beta) e^{j\beta\xi} d\beta \quad (\text{A.22})$$

or, since $V_m(\beta) = j\omega u(\beta)$, where u is the displacement,

$$\frac{p_{sm}}{D^*} = \frac{j}{2\pi^2 a} \left[\frac{m\pi}{a} \right]^2 \left[\frac{\omega^2 \rho_o}{D^*} \right] \int_{-\infty}^{+\infty} I_m(\beta) u_m(\beta) e^{-j\beta\xi} d\beta \quad (\text{A.23})$$

From the above expressions, the scattered pressure component can be evaluated.



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